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Parameters for Quantitative Comparison of Two-, Three-, and Four-level Laser Media, Operating Wavelengths, and Temperatures

by Jeffrey O. White

ARL-RP-0296 **August 2010**

A reprint from the *IEEE J. Quantum Electronics*, vol. 45, no. 10, October 2009.

NOTICES

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Adelphi, MD 20783-1197

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REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED (From - To)**
August 2010	Reprint	

4. TITLE AND SUBTITLE	
Parameters for Quantitative Comparison of Two-, Three-, and Four-level Laser Media, Operating Wavelengths, and Temperatures	**5a. CONTRACT NUMBER**
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)	
Jeffrey O. White	**5d. PROJECT NUMBER**
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	**8. PERFORMING ORGANIZATION REPORT NUMBER**
U.S. Army Research Laboratory ATTN: RDRL-SEE-O 2800 Powder Mill Road Adelphi, MD 20783-1197	ARL-RP-0296

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES

A reprint from the *IEEE J. Quantum Electronics*, vol. 45, no. 10, October 2009.

14. ABSTRACT

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15. SUBJECT TERMS

Erbium lasers, holmium lasers, laser physics, neodymium lasers, quasi-four-level lasers, quasi-three-level laser, quasi-two-level lasers, ytterbium lasers

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**	UU	14	Jeffrey O. White
Unclassified	Unclassified	Unclassified			**19b. TELEPHONE NUMBER** (*Include area code*) (301) 394-0069

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.1

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Jeffrey Owen White, *Member, IEEE*

***Abstract*—Several parameters are proposed for describing the statistical thermodynamic component of the exchange of photons between a pump and a laser beam. They are based on the occupation probability of absorbing and emitting, pump and laser levels, and are complementary to the optical cross sections. The "occupation factor," f_0, is appropriate for describing an optical amplifier in the small signal regime. f_1 is appropriate for describing an amplifier in the large signal regime, e.g., a laser. They serve to facilitate a quantitative comparison of laser gain media, operating temperatures, and choice of pump and laser wavelengths. After a simple scaling, both occupation factors have a numerical value that coincides well, in most cases, with conventional usage of the terms two-, three-, and four-level laser. They can thus serve as an unambiguous, quantitative alternative to the quasi-two-, quasi-three-, and quasi-four-level terminology. The proposed definitions are general enough to apply to many types of gain media, but are particularly useful for comparing systems with discrete levels, pumped with a narrowband source, in near-resonance with the laser wavelength. Several low-quantum-defect combinations of pump and laser wavelengths are analyzed for Er^{3+}, Nd^{3+}, Yb^{3+}, and Ho^{3+} in YAG, as a function of temperature.**

***Index Terms*—Erbium lasers, holmium lasers, laser physics, neodymium lasers, quasi-four-level lasers, quasi-three-level lasers, quasi-two-level lasers, ytterbium lasers.**

I. Introduction

The terms "three-level-system," and "four-level-system" have been in wide use ever since the invention of the laser, and were used to describe masers even before that time. The introduction of a "system level" parameter that varies continuously from two to four would facilitate a quantitative comparison of laser media, operating temperatures, and choice of pump and laser transitions. The parameters proposed below, based on the occupation probability of absorbing and emitting pump and laser levels, can serve that purpose.

The "quasi-level" terminology is also in wide use. Counting its occurrence in the title and abstract alone, 32 papers used "quasi-two, three-, or four-level" in 2007. In the ten years from 1997 to 2006, the terminology was used in the title or abstract of 197 papers [1]. The terminology appears in the body of a far larger number of papers. Quasi-three-level is particularly ambiguous, because it could refer to systems that are either better or worse than "three-level." The aim of this paper is to introduce a more quantitative terminology, useful to the laser designer for comparing various media, choices of pump and laser wavelengths, and different operating temperatures.

The proposed terminology is based upon parameters, e.g., the occupation factor, f_0, which is a component of the small-signal gain of an optical amplifier. The occupation factor, f_1, is a component of the large-signal coupling between a pump and a signal in, e.g., a laser. f_0 and f_1 can be simply scaled to yield "system level" parameters ℓ_0 and ℓ_1 that vary continuously between zero and four. By "system" we mean a combination of ion (or other species with well-defined levels), pump and laser transitions, and temperature. The numerical values of ℓ_0 and ℓ_1 are close to three for most systems that would commonly be called three-level, and close to four for most systems that would commonly be called four-level [2].

These parameters should be most useful in the study of lasers: 1) that have a low quantum defect; 2) that are pumped with narrow band light; 3) where ground state absorption at the laser wavelength is a factor; or 4) where stimulated emission at the pump wavelength is a factor. Such is the case for diode-pumped lasers based on Er^{3+}, Nd^{3+}, Yb^{3+}, and Ho^{3+}, which have been variously described as quasi-two-level, quasi-three-, or quasi-four, depending on the particular transitions and temperature.

Like the optical cross section, the occupation factors f_0, and f_1 indicate the suitability of a gain center, e.g., Er^{3+} in YAG, for an amplifier or laser, as a function of temperature, pump and laser wavelengths. Whether or not a particular transition will be a practical laser depends on additional extrinsic factors, e.g., doping density, pump intensity, mirror reflectivities, etc. The point in this paper is to isolate the statistical thermodynamic aspect of the ion or atom, which depends explicitly on temperature and energy level alignments. In the next sections, we define the occupation factors and system-level parameters, and apply them to Er^{3+}, Nd^{3+}, Yb^{3+}, and Ho^{3+} in YAG.

II. Occupation Factors

A. Background

In a classic four-level laser [Fig. 1(a)], in steady state, the relaxation of electrons from the upper pump level to the upper laser level is fast compared to the thermal excitation of electrons in the reverse direction, and typically fast compared to the rates of optical absorption and emission. Therefore, the population in the upper laser level is large compared to the upper pump level, favoring the stimulated emission of laser photons over pump photons.

Manuscript received November 03, 2008; revised February 02, 2009 and March 10, 2009. Current version published September 23, 2009.

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Digital Object Identifier 10.1109/JQE.2009.2020607

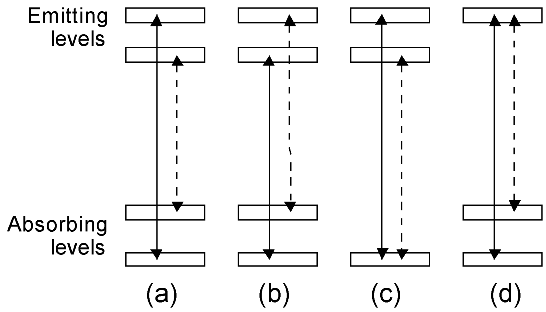

Fig. 1. Different possibilities for four- and three-level systems, showing pump (solid) and laser (dashed) transitions. All four cases can be quantitatively compared, on the basis of the "system level", ℓ.

The preferred energy level occupancy in the lower levels is the opposite, favoring the absorption of pump photons over laser photons. Compared to three-level systems, four-level systems are easy to invert, because the upper laser level is easily populated, and the lower laser level is rapidly depopulated.

In a classic three-level system [Fig. 1(c)], the lower laser level coincides with the ground state, e.g., in $Cr:Al_2O_3$ at 694.3 nm, in which case the unexcited medium absorbs at λ_{L}. Alternatively, the upper pump level can coincide with the upper laser level [Fig. 1(d)], in which case the absorption at λ_{P} saturates very easily.

In two-level systems, the initial state for emission of pump and laser photons is shared, as well as the initial state for absorption. These systems can be inverted in steady state only if the upper level has a higher degeneracy. We defer further consideration of degeneracy, because it will not change the substance of the conclusions below.

B. Derivation of Occupation Factors f_0 and f_1

Consider the propagation of light at λ_{P} and λ_{L}, coupled by a gain medium held at constant temperature. We group together the absorbing states that are close to the ground state, compared to the photon energies. They could be the lower manifold of Er^{3+}, for example, or the lower two manifolds of Nd^{3+}. Absorbing states within $\mathrm{k_B T}$ of the ground state can obviously be thermally populated.

Also grouped together are the highly excited states, i.e., those which can emit photons. States within $\mathrm{k_B T}$ of the lowest emitting state can also be thermally populated. The rate equations for the total population density of absorbing states (N_1) and the total population of emitting states (N_2), at any point in the gain medium, include absorption and emission at both wavelengths are [3]

$$\begin{aligned}\frac{dN_1}{dt} =& +\Phi_P\sigma_P(f_{eP}N_2 - f_{aP}N_1)\\ &+\Phi_L\sigma_L(f_{eL}N_2 - f_{aL}N_1) + N_2W_{21} \quad (1)\\ \frac{dN_2}{dt} =& -\frac{dN_1}{dt} \quad (2)\end{aligned}$$

where Φ_P (Φ_L) is the pump (laser) photon flux, σ_P (σ_L) is the absolute cross section at the pump (laser) wavelength, and W_{21} is the spontaneous emission rate. The total number of ions $N_{\mathrm{tot}} = N_1 + N_2$ is constant. f_{eL} is the probability that the electron is in the initial state for emitting a Laser photon, given that it is in one of the emitting states. f_{aP} is the probability that the electron is in the initial state for absorbing a Pump photon, given that it is in one of the absorbing states.

Neglecting the contribution of (isotropic) spontaneous emission, colinear laser and pump beams will propagate according to

$$\frac{d\Phi_L}{dz} = \sigma_L(f_{eL}N_2 - f_{aL}N_1)\Phi_L \quad (3)$$

$$\frac{d\Phi_P}{dz} = \sigma_P(f_{eP}N_2 - f_{aP}N_1)\Phi_P. \quad (4)$$

Solving (1), (2) in steady state for N_1 and N_2, and substituting into (3) and (4), we obtain

$$\frac{d\Phi_L}{dz} = \frac{\sigma_L\sigma_P N_{tot}(f_{eL}f_{aP} - f_{aL}f_{eP} - f_{aL}W_{21}/\Phi_P\sigma_P)}{\Phi_P\sigma_P(f_{eP}+f_{aP}) + \Phi_L\sigma_L(f_{aL}+f_{eL}) + W_{21}}\Phi_L\Phi_P \quad (5)$$

$$\frac{d\Phi_P}{dz} = -\frac{d\Phi_L}{dz}. \quad (6)$$

When $\Phi_P\sigma_P, \Phi_L\sigma_L \gg W_{21}$, we have

$$\frac{d\Phi_L}{dz} = \frac{\sigma_L\sigma_P N_{tot}(f_{eL}f_{aP} - f_{aL}f_{eP})}{\Phi_P\sigma_P(f_{eP}+f_{aP}) + \Phi_L\sigma_L(f_{aL}+f_{eL})}\Phi_L\Phi_P. \quad (7)$$

The laser photon flux increases at the expense of the pump beam, even when the pump is the weaker of the two, if $\Lambda > 0$, where

$$\Lambda \equiv f_{eL}f_{aP} - f_{aL}f_{eP}. \quad (8)$$

If $\Lambda < 0$, the opposite occurs, because (1)–(4) are symmetric with respect to interchange of pump and laser.

In special cases, (7) can be separated into a product of cross section and concentration, and a factor involving just occupancy. In an amplifier, for example, when $\Phi_L\sigma_L(f_{aL} + f_{eL}) \ll \Phi_P\sigma_P(f_{eP} + f_{aP})$, Φ_{L} is effectively uncoupled from Φ_{P}, and grows exponentially according to

$$\frac{d\Phi_L}{dz} = \sigma_L N_{tot}\frac{f_{eL}f_{aP} - f_{aL}f_{eP}}{f_{aP}+f_{eP}}\Phi_L. \quad (9)$$

The part of the small-signal gain coefficient that depends on occupancy is given by

$$f_0 \equiv \frac{f_{eL}f_{aP} - f_{aL}f_{eP}}{f_{aP}+f_{eP}} \quad (10)$$

Far above threshold in a laser cavity, one may have $\Phi_L\sigma_L(f_{aL} + f_{eL}) \gg \Phi_P\sigma_P(f_{eP} + f_{aP})$. In this case, the laser flux still grows if $\Lambda > 0$, but the gain is no longer exponential, and the laser is strongly coupled to the pump according to

$$\frac{d\Phi_L}{dz} = \sigma_P N_{tot}\frac{f_{eL}f_{aP} - f_{aL}f_{eP}}{f_{eL}+f_{aL}}\Phi_P. \quad (11)$$

The part of the coupling coefficient that depends on occupancy is given by

$$f_1 \equiv \frac{f_{eL}f_{aP} - f_{aL}f_{eP}}{f_{eL}+f_{aL}}. \quad (12)$$

In the symmetric case, where $\Phi_L \sigma_L = \Phi_P \sigma_P$, the relevant occupation factor is

$$f_2 \equiv 2 \frac{f_{eL} f_{aP} - f_{aL} f_{eP}}{f_{eL} + f_{aP} + f_{aL} + f_{eP}}. \qquad (13)$$

Thus different combinations of the occupancy factors are figures of merit in different situations. If the pump and laser wavelengths are interchanged, the new occupancy factors f_i' are given by

$$f_0' = -f_1, \quad f_1' = -f_0 \quad \text{and} \quad f_2' = -f_2. \qquad (14)$$

In what follows, we will show that the occupation factors furnish a direct link with the widely used three-level- and four-level-laser terminology.

C. Connection With "System Level"

In this section, we will show that the quantities

$$\ell_i = 2(f_i + 1) \qquad (15)$$

have numerical values close to three in cases where we expect the medium to have three-level character, and close to four when we expect the medium to have four-level character.

In the ideal four-level case, $f_{eL} = f_{aP} = 1$ and $f_{eP} = f_{aL} = 0$, therefore $f_0 = f_1 = 1$ and $\ell_0 = \ell_1 = 4$. In the three-level case of Fig. 1(c), the optimum occupancies would be $f_{eL} = f_{aP} = f_{aL} = 1$, and $f_{eP} = 0$. In the strong laser case ($f_1 = 1/2$, $\ell_1 = 3$), ground state absorption at λ_{L} will be a factor, and we can expect the medium to have three-level character. In the strong pump case ($f_0 = 1$, $\ell_0 = 4$), ground state absorption at λ_{L} will be less of a factor, and the medium will have four-level character.

In the three-level case of Fig. 1(d), the optimum occupancies would be $f_{eL} = f_{aP} = f_{eP} = 1$ and $f_{aL} = 0$. In the strong pump case ($f_0 = 1/2$, $\ell_0 = 3$), transitions at λ_{P} are rapid compared to those at λ_{L}, the absorption at λ_{P} will saturate, and we can expect the medium to have three-level character. In the strong laser case ($f_1 = 1$, $\ell_1 = 4$), absorption saturation at λ_{P} will be less of a factor, and the medium will have four-level character.

In the two-level case, the only optimum scenario is $f_{eL} = f_{eP} = f_{aP} = f_{aL} = 1$, therefore $f_0 = f_1 = 1$ and $\ell_0 = \ell_1 = 2$.

In conventional usage, a three-level system does not become a two- or four-level system as the population shifts between N_1 and N_2. The ℓ_{i} satisfy this criterion because of the way f_{eL}, etc. are defined. As they should, the ℓ_{i} depend on which of the two wavelengths is considered the laser. If the pump and laser wavelengths are interchanged, the new system levels ℓ_i' are given by

$$\ell_0 + \ell_1' = \ell_0' + \ell_1 = 4. \qquad (16)$$

D. Alternative Definitions of "System Level,"

The occupation probabilities can be combined in other ways in an attempt to quantify the "system level" concept. For example, we can premise that the level that absorbs (emits) pump photons contributes one to ℓ when it is full (empty) relative to the other absorbing (emitting) states, etc.

$$\ell_3 = f_{eL} + (1 - f_{eP}) + f_{aP} + (1 - f_{aL}). \qquad (17)$$

A reasonable normalization to consider would be

$$\ell_4 \equiv \frac{f_{eL}}{f_{eL} + f_{eP}} + \left(1 - \frac{f_{eP}}{f_{eL} + f_{eP}}\right) + \frac{f_{aP}}{f_{aP} + f_{aL}} + \left(1 - \frac{f_{aL}}{f_{aP} + f_{aL}}\right), \qquad (18)$$

or equivalently

$$\frac{\ell_4 - 2}{2} = \frac{f_{eL} f_{aP} - f_{eP} f_{aL}}{(f_{eL} + f_{eP})(f_{aP} + f_{aL})}. \qquad (19)$$

Due to the normalization in (18) and (19), one can have $\ell_4 \sim 4$ even when the initial states for absorbing at λ_{P} and emitting at λ_{L} are high in their respective manifolds, in which case the pump-laser coupling vanishes according to (9) and (11). If pump and laser wavelengths are interchanged, the new system level is given by $\ell_4 + \ell_4' = 4$.

If the levels in the emitting manifold are in thermal equilibrium, and the levels in the absorbing manifold are in thermal equilibrium, ℓ_4 depends only on the difference between the pump and laser energy levels. In contrast, ℓ_0 and ℓ_1 depend on the entire (populated) level structure. ℓ_3 and ℓ_4 lack the clear connection to intuitive physical quantities, e.g., the small signal gain and large signal coupling, so we do not consider them any further.

III. Quasi-Two-, Quasi-Three-, and Quasi-Four-Level Lasers

Here, the system level is calculated for several ions of interest and several choices of transitions. The energy levels are obtained from [4]. We assume that the occupation probability for a sublevel within the emitting states follows a Boltzmann distribution. For example

$$f_{eL} = \exp(-E_{eL}/kT) \Big/ \sum_e \exp(-E_e/kT) \qquad (20)$$

where the sum is over all the emitting states. The same assumption is made for the absorbing states. The system levels thus calculated at 300 K are summarized in Table I.

TABLE I
System Level ℓ, Calculated for Various Gain Media, and Wavelengths, at 300 K

	λ_{P} (nm)	λ_{L} (nm)	ℓ_0	ℓ_1
Er:YAG	1470	1645	2.2	2.5
"	1532	1645	2.2	2.4
Er:Sc_2O_3	1535	1558	2.1	2.1
Nd:YAG	808	1064.1	2.8	2.9
"	808	946	3.2	2.9
"	869	946	2.6	2.9
"	884	946	2.4	2.3
"	886	946	2.3	2.5
Yb:YAG	941	1030	3.3	3.6
"	968	1030	2.8	3.6
Ho:YAG	1907	2097	2.1	2.2

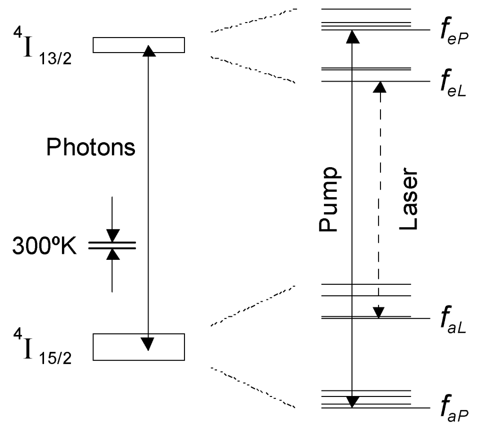

Fig. 2. Er^{3+} ground state manifold and first excited state manifold. The scale on the right is magnified.

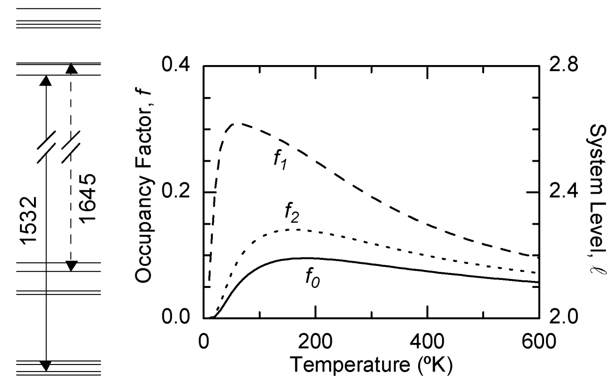

Fig. 4. Er:YAG, $\lambda_{\mathrm{P}} = 1532$ nm, $\lambda_{\mathrm{L}} = 1645$ nm: (a) energy levels, pump and laser transitions, (b) temperature dependence of the occupancy factors and corresponding system levels.

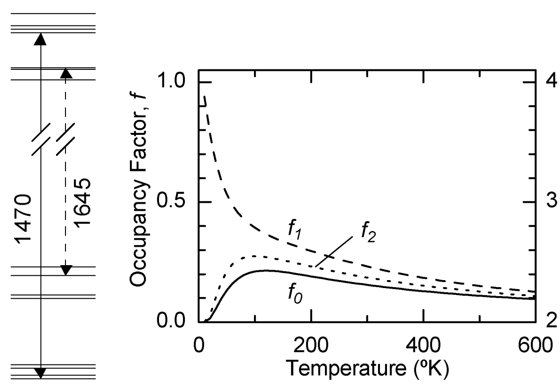

Fig. 3. Er:YAG, $\lambda_{\mathrm{P}} = 1470$ nm, $\lambda_{\mathrm{L}} = 1645$ nm: (a) energy levels, pump transition (solid line), and laser transition (dashed line) and (b) temperature dependence of the occupancy factors and corresponding system levels.

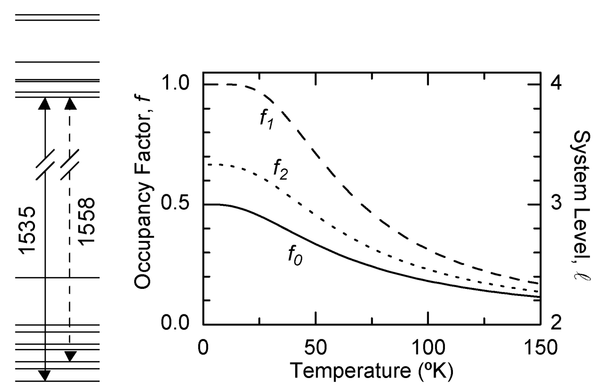

Fig. 5. $Er:Sc_2O_3$, $\lambda_{\mathrm{P}} = 1535$ nm, $\lambda_{\mathrm{L}} = 1558$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

A. Er:YAG

In Er^{3+}, the *laser* transitions in the 1.6-μm region are between the $^4I_{15/2}$ (ground state) manifold and the $^4I_{13/2}$ (first excited state) manifold (Fig. 2). For low-quantum-defect applications, the *pump* transitions can also be between the same two manifolds. In the absence of upconversion, these are the only two manifolds that have significant occupation. Consideration of cross sections leads to pumping at 1470 nm, or 1532 nm and lasing at 1617 or 1645 nm [5], [6]. Both laser wavelengths behave similarly; for purposes of illustration, we focus on 1645 nm.

For $\lambda_{\mathrm{P}} = 1470$ nm and $\lambda_{\mathrm{L}} = 1645$ nm, the level alignment is favorable in both the absorbing states and the emitting states (Fig. 3), i.e., the lower laser level and the upper pump levels are high in their respective manifolds, and the upper laser level and lower pump levels are low in their respective manifolds. Neglecting the changes in the level energies, we can easily plot the occupation factors as a function of temperature (Fig. 3). The right hand axis shows the corresponding system levels. In the large signal case, the system level ℓ_1 increases from 2.3 at 600 K to 4.0 at 0 K. Other factors being equal, there is a clear advantage to operating such a laser below 10 K. In the small signal case, a maximum of $\ell_0 = 2.4$ is reached at 120 K. Other factors being equal, this would be the optimum temperature to operate an amplifier. At lower temperatures, the population of the upper laser level goes to zero, therefore f_0 and the small signal gain go to zero, ℓ_0 goes to two, and the system acquires two-level character.

For $\lambda_{\mathrm{P}} = 1532$ nm and $\lambda_{\mathrm{L}} = 1645$ nm, the level alignment is favorable in the absorbing states, but not in the emitting states (Fig. 4). At low temperature, both the upper laser level and the lower pump level freeze out, therefore the f_i go to zero. The maximum value of $\ell_0 = 2.2$ occurs at $\sim$180 K. The maximum value of $\ell_1 = 2.6$ occurs at 60 K. Of course, the absorption and emission cross sections will also change with temperature. The optimum temperature for lasing would fall in the range 60–180 K if all other factors were constant.

An ultra-low quantum defect laser has recently been demonstrated at 77 K in $Er:Sc_2O_3$ [7]. For $\lambda_{\mathrm{P}} = 1535$ nm and $\lambda_{\mathrm{L}} = 1558$ nm, the level alignment is favorable in the absorbing states, however, the two emitting states coincide (Fig. 5). At 80 K, $\ell_0 \sim 2.5$ and $\ell_1 \sim 2.9$. In the large signal case, it would be necessary to operate below 25 K to obtain four-level character.

B. Nd:YAG

For Nd:YAG, we take the absorbing states to be a combination of the ground state $^4I_{9/2}$ manifold and the $^4I_{11/2}$; a total of eleven levels. We take the emitting states to be the metastable $^4F_{3/2}$ manifold, the $^4F_{5/2}$, and the $^2H_{9/2}$; a total of ten levels.

For $\lambda_{\mathrm{P}} = 808$ nm and $\lambda_{\mathrm{L}} = 1064.1$ nm, all five manifolds are involved (Fig. 6). The occupancy factors f_0, f_1, and f_2 are suppressed because the population in the $^4F_{3/2}$ manifold is divided

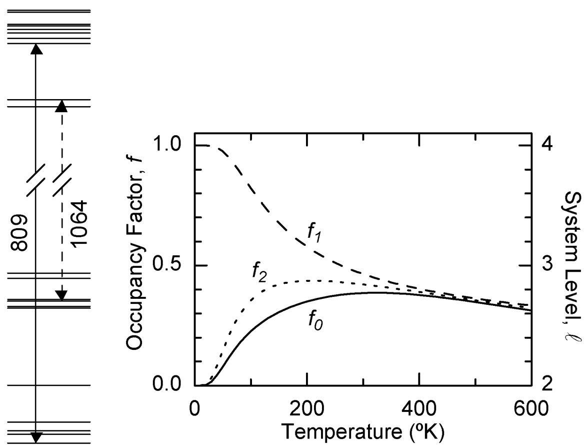

Fig. 6. Nd:YAG, $\lambda_{\mathrm{P}} = 809$ nm, $\lambda_{\mathrm{L}} = 1064.1$ nm: (a) energy levels, pump and laser transitions, (b) temperature dependence of the occupancy factors and corresponding system levels.

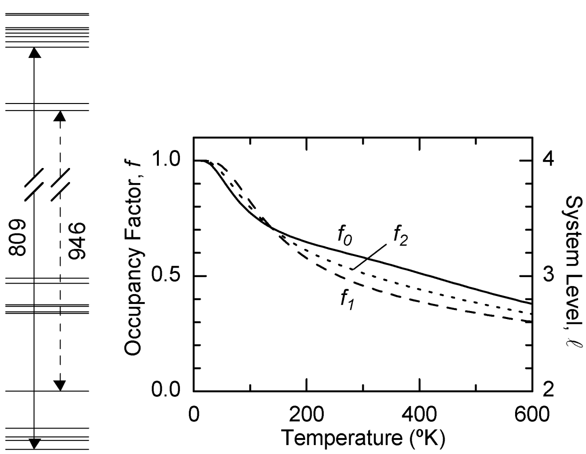

Fig. 7. Nd:YAG, $\lambda_{\mathrm{P}} = 809$ nm, $\lambda_{\mathrm{L}} = 946$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

between two closely spaced levels. Because the upper laser level is not the lowest metastable level, f_0 goes to zero as the temperature goes to zero.

946 nm can be made to lase by suppressing the 1064 nm emission (Fig. 7) [8]–[18]. Unlike at 1064 nm, laser emission at 946 nm begins from the lowest level in the $^4F_{3/2}$ manifold, therefore ℓ_0 and ℓ_2 increase to four at low temperature, as well as ℓ_1.

To lower the quantum defect, it is possible to pump *and* lase between the $^4I_{9/2}$ and $^4F_{3/2}$ manifold. For $\lambda_{\mathrm{P}} = 869$ nm, and $\lambda_{\mathrm{L}} = 946$ nm [19], the initial states for pump absorption and laser emission are the lowest lying sublevels in their respective manifold (Fig. 8), therefore the system acquires four-level character at low enough temperature. Because of the small splitting in the $^4F_{3/2}$ manifold, the temperature has to be below ~20–50 K for this to happen.

When $\lambda_{\mathrm{P}} = 884.25$ nm [20], the quantum defect is only slightly lower, but the low temperature behavior is very different because the initial state for pump absorption becomes frozen out (Fig. 9). ℓ_0 has an maximum value of ~2.4 at 240 K. ℓ_1 has an maximum value of ~2.4 at 390 K.

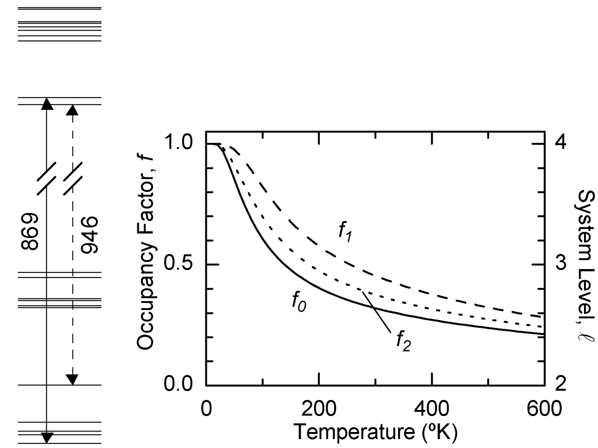

Fig. 8. Nd:YAG, $\lambda_{\mathrm{P}} = 869$ nm, $\lambda_{\mathrm{L}} = 946$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

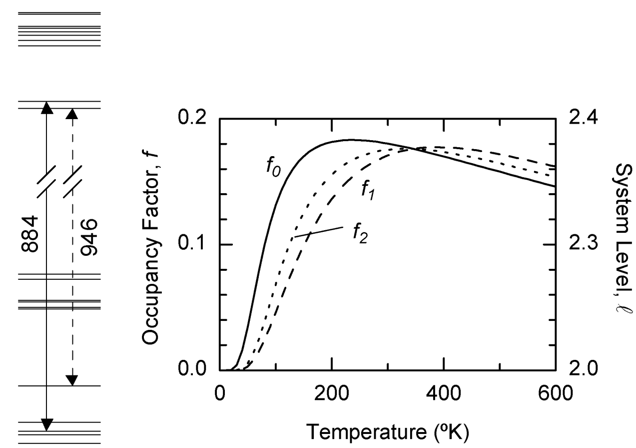

Fig. 9. Nd:YAG, $\lambda_{\mathrm{P}} = 884$ nm, $\lambda_{\mathrm{L}} = 946$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

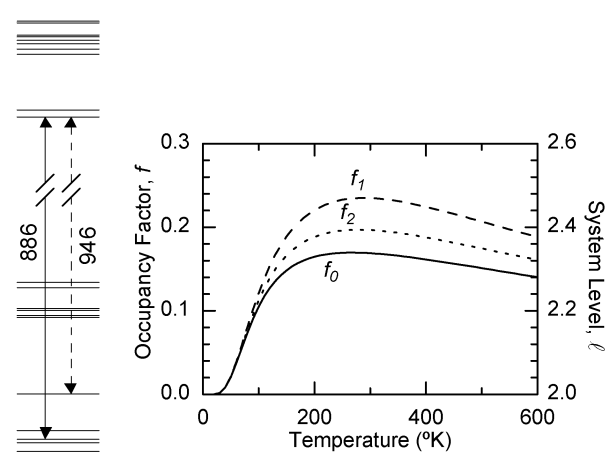

Fig. 10. Nd:YAG, $\lambda_{\mathrm{P}} = 886$ nm, $\lambda_{\mathrm{L}} = 946$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

When $\lambda_{\mathrm{P}} = 886$ nm [20], the upper pump and laser levels coincide, so the system levels never rise above three (Fig. 10).

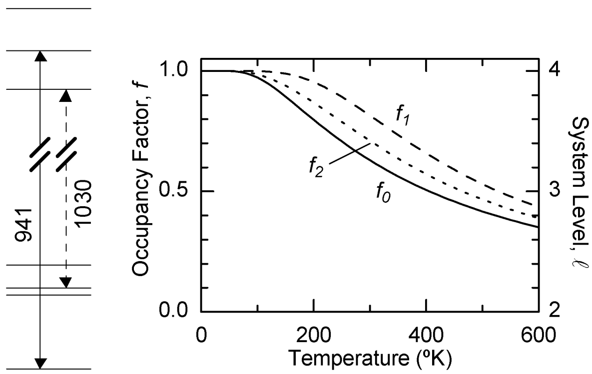

Fig. 11. Yb:YAG, $\lambda_{\mathrm{P}} = 941$ nm, $\lambda_{\mathrm{L}} = 1030$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

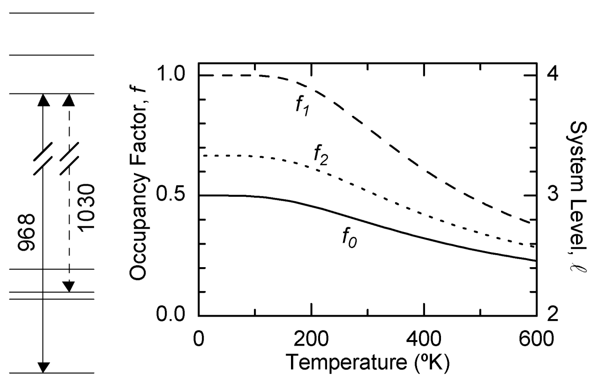

Fig. 12. Yb:YAG, $\lambda_{\mathrm{P}} = 968$ nm, $\lambda_{\mathrm{L}} = 1030$ nm: (a) energy levels, pump and laser transitions, (b) temperature dependence of the occupancy factors and corresponding system levels.

ℓ_0 has an maximum value of ~2.3 at 240 K. ℓ_1 has an maximum value of ~2.5 at 390 K.

C. Yb:YAG

In Yb:YAG the $^2F_{7/2}$ and $^2F_{5/2}$ manifolds are the only energy levels involved in $4f - 4f$ transitions. The low quantum defect and the availability of diode laser pumping are well-known advantages in this system [21]–[33].

When $\lambda_{\mathrm{P}} = 941$ nm and $\lambda_{\mathrm{L}} = 1030$ nm, the level alignments are favorable in both the absorbing and emitting states, and the initial states for pump absorption and laser emission are the lowest in their respective manifolds (Fig. 11). The splittings are such that the system has nearly ideal four-level character at temperatures below ~100–200 K.

When $\lambda_{\mathrm{P}} = 968$ nm and $\lambda_{\mathrm{L}} = 1030$ nm, the two emitting levels coincide, reducing the small-signal system level to a maximum of $\ell_0 \sim 3$ (Fig. 12). In the large signal case, ℓ_1 rises to 4 at temperatures below 200 K.

D. Ho:YAG

In Ho:YAG, it is possible to pump and lase between the $^5I_8$ and $^5I_7$ manifolds [34]. When $\lambda_{\mathrm{P}} = 1907$ nm and $\lambda_{\mathrm{L}} = 2097$ nm, the level alignments are favorable in both the absorbing and emitting states, and the initial states for pump absorption and laser emission are the lowest in their respective manifolds (Fig. 13). Due to the close spacings within the upper manifold, the system level approaches four only at ~5 K.

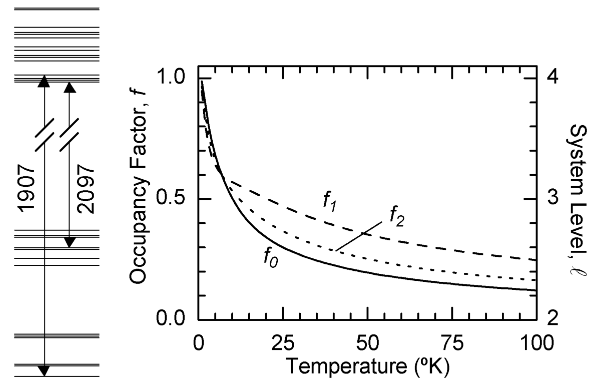

Fig. 13. Ho:YAG, $\lambda_{\mathrm{P}} = 1907$ nm, $\lambda_{\mathrm{L}} = 2097$ nm: (a) energy levels, pump and laser transitions and (b) temperature dependence of the occupancy factors and corresponding system levels.

IV. Discussion

The search for systems with a low quantum defect inevitably leads to a departure from the ideal case of a four-level-system. At some point, either ground state absorption at λ_{L} will become a factor, or absorption saturation at λ_{P}, or both. Often, the systems lie in between the classic two-level, three-level, and four-level cases. To quantitatively compare the different ions, transitions, and temperatures, we have introduced the occupation factors f_i, and the associated system levels ℓ_{i}.

Using eleven pump and laser transitions in well-known rare-earth ions as examples, one can see that ℓ_0 and ℓ_1 span the range from two to four (Fig. 14). The systems that suffer from thermal population of the lower laser level or the upper pump level have higher level parameters at 80 K. The systems that rely on thermal excitation to populate the lower pump level or the upper laser level may have higher level parameters at 300 K.

The calculations of ℓ presented here assume that the energy level structure between 0–600 K is close to that measured at 300 K. Continuous energy level data for Er^{3+}, Nd^{3+}, Yb^{3+}, and Ho^{3+} from 0 to 600 K are not available. The energy levels are typically determined at only a few discrete temperatures, e.g., 4.2 K, 77 K, and 300 K [4]. In these cases, the energies shift by only a few cm^{-1}.

The present work can be compared to several previous steady-state treatments. For example, an early model included the factors f_{aL} and f_{eL} in a calculation of threshold and output power as a function of temperature [8]. The broadband regime has been considered, where the pump induced transitions between all states of two manifolds [35]. A theory which takes into account ground state depletion and gain saturation was used to calculate the optimum output coupler reflectivity [24]. Another model does not include saturation at λ_{P}, but does include an arbitrary distribution of pump and laser spatial modes [25]. A rate equation analysis considered the propagation of Gaussian pump and laser beams, taking into account the effects of absorption saturation, temperature profile, and beam quality factor of the pump diode [36]. The factor Λ has been included in a calculation of laser output, optimum output mirror reflectivity, crystal length, doping and temperature [28].

For historical reasons related to large-quantum-defect pumping with arc lamps, and the early use of Nd^{3+} and Cr^{3+}, the distinction between three- and four-level lasers focused on

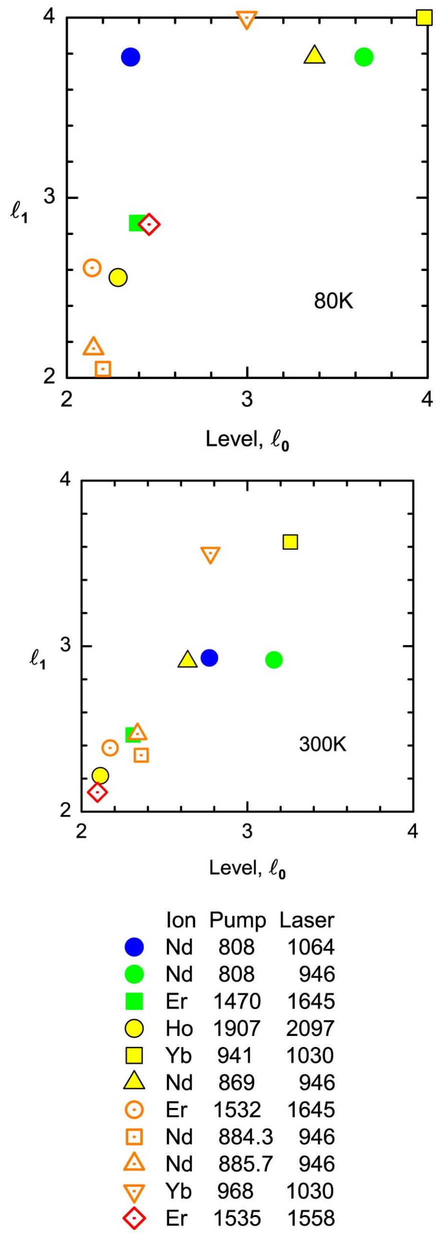

Fig. 14. ℓ_1 vs. ℓ_0 for a variety of known lasers at (a) 80 K and (b) 300 K. The color is keyed to the quantum defect. The host material is YAG in all cases, except in the last (red) case, where the host is Sc_2O_3.

the splitting of the absorbing levels [2]. In this work, we wish to emphasize the equal *physical* importance of the extraneous levels and the splitting of the emitting levels. Emitting states that are too close in energy leads to saturation of pump absorption, and lowering of the gain/coupling because electrons can be easily thermally excited out of the upper laser level. The splitting of the emitting states has become of equal *technological* importance with the advent of low-quantum-defect pumping with diode lasers.

More recently, the parameter $\gamma = 1 + f_{aL}/f_{eL}$ has been used to describe the transition from three-level- to four-level-lasers [37]. When one ion in the upper laser level transitions to the lower laser level, the population inversion decreases by γ. The parameters proposed in this work, f_0, f_1, ℓ_0, and ℓ_1, depend on all four level occupancies, are simply related to intuitive figures of merit, e.g., the small-signal gain, and can describe the complete range from two-level to four-level lasers.

Nd^{3+} presents an interesting case when $\lambda_{\mathrm{P}} = 808$ nm and $\lambda_{\mathrm{L}} = 1064.1$ nm. This would typically be called a four-level laser, however $\ell_1 = 2.9$. Several points can be made concerning this discrepancy.

1) A low value of ℓ_1 does not rule out making a good laser, if the doping concentration and cross section are high enough to compensate.
2) In cw, the population in the R_2 level is clearly reduced by the presence of the R_1 level. Therefore, Nd deserves a lower figure of merit, compared for example to Yb with $\lambda_{\mathrm{P}} = 941$ nm and $\lambda_{\mathrm{L}} = 1030$ nm.
3) The presence of the nearby $\mathrm{R}_2 - \mathrm{Y}_2$ transition at 1064.4 nm will increase the effective gain at 1064.1 nm, to the extent that the transitions overlap.
4) Additional output power can be obtained if multiple transitions are simultaneously lasing, e.g., the $\mathrm{R}_2 - \mathrm{Y}_3$ and $\mathrm{R}_1 - \mathrm{Y}_2$ transitions.
5) In the pulsed regime, it is well known that energy stored in the R_1 level can be extracted following thermal excitation in the the R_2 level. However, in steady state, based on our assumption of emitting states in thermal equilibrium, the transitions from R_1 to R_2 are very nearly balanced by the transitions from R_2 to R_1.
6) One could define a level parameter, e.g., ℓ_4, that has a value close to four for Nd^{3+}, but it does not have the clear connection with intuitive physical quantities (like the small-signal gain, and large-signal coupling-to-the-pump) that ℓ_0 and ℓ_1 have.

Although the definitions of the f_i and ℓ_i do not depend on the absorbing (or emitting) levels being in thermal equilibrium, the calculations presented in Section III for illustration do assume Boltzmann distributions for each population. The limits of validity can be estimated by comparing the optical transition rate (connecting absorbing to emitting states) to the nonradiative transition rate (among absorbing or emitting states). The optical transition rate induced by the pump or the laser can be estimated with $W_r = I\sigma f/h\nu$, where f is the occupancy of the relevant initial state.

In resonantly pumped Er:YAG at 300 K, the absorbing states are all located within a single manifold, and the emitting states as well. Therefore, the level spacings are relatively small and the intra-manifold non-radiative transition rate can be estimated from the $\sim 1.4 \times 10^{11}$ Hz width of the 1532 nm absorption peak. At 1532 nm, the pump intensity should satisfy $I_P < 8 \times 10^{11}$ W/cm^2 if the $^4\mathrm{I}_{15/2}$ manifold is to have a thermal distribution. At 1645 nm, the laser intensity should satisfy $I_L < 5 \times 10^{12}$ W/cm^2.

In Nd:YAG, for $\lambda_{\mathrm{P}} = 809$ nm and $\lambda_{\mathrm{L}} = 1064.1$ nm, the constraint is more severe because the absorbing states comprise two manifolds with a large splitting, reducing the inter-manifold nonradiative transition rate. The $^4\mathrm{I}_{11/2}$ lifetime at room temperature [38] corresponds to $\sim 6 \times 10^9$ Hz. At 809 nm, the pump intensity should therefore satisfy $I_P < 7 \times 10^{10}$ W/cm^2 in order that one thermal distribution be maintained across both manifolds. At 1064.1 nm, the laser intensity should satisfy $I_L < 5 \times 10^9$ W/cm^2.

V. Conclusion

The "two-, three-, and four-level system" terminology is widely used to describe lasers. The terminology suffices if the levels are well-separated compared to $\mathrm{k_B T}$, or if some levels completely overlap. In low-quantum-defect systems, intermediate cases arise because of partial thermal excitation of the lower laser level or the upper pump level. If the pumping involves only a single transition, and the lasing as well, there is a straightforward definition of system level, based on energy level structure and temperature, that spans the range from two to four. ℓ_0 and ℓ_1 describe the system under small- and large-signal conditions. They are figures of merit for a gain medium, complementary to the cross section. They are expected to be useful for comparing systems: 1) that have a low quantum defect, i.e., they are pumped nearly in resonance with the laser wavelength; 2) that are pumped with narrow band light; 3) where ground state absorption at the laser wavelength is a factor; or 4) where stimulated emission at the pump wavelength is a factor. Like the optical cross section, the occupation factors f_0, and f_1 help in the design of a laser or amplifier by facilitating the comparison of different gain centers, choice of pump and laser wavelengths, and operating temperature.

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